Resistance Networks
Information Sheet

This sheet is to help you develop a basic understanding of how to solve resistance networks and covers the four basic operations.

(1) Resistances in series
(2) Resistances in parallel
(3) Combinations in parallel and in series with a single path
(4) General resistance networks through nodal analysis

If you remember the Temperature difference (ΔT) is the equivalent of the Voltage difference, the heat flow (Q)is the equivalent of the current and the thermal resistance (R) is related to them by;

$$R = \frac{\Delta T}{Q}$$

(1) Resistances in Series

When you have two resistances in series, the total resistance is the sum of the resistances. For example consider Figure (1).

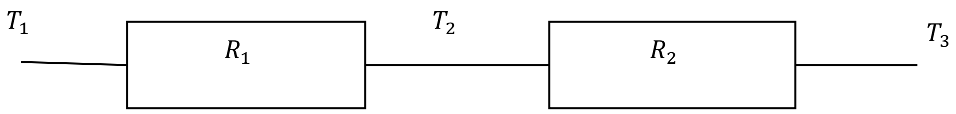

Figure 1: Two resistors in series

The heat flow through the first resistor must be the same as the heat flow through the second resistor (otherwise you are not conserving energy). Therefore if we use 1 and 2 to indicate the flow through the two resistors and no subscript to indicate the total resistance;

$$Q_1 = Q_2 = \frac{T_2 - T_1}{R_1} = \frac{T_3 - T_2}{R_2} = Q = \frac{T_3 - T_1}{R}$$

Or

$$R = \frac{T_3 - T_1}{Q} = \frac{T_3 - T_2}{Q} + \frac{T_2 - T_1}{Q} = R_1 + R_2$$

Therefore the resistance (R) is the sum of the two resistances ($R_1 + R_2$).

(2) Resistances in Parallel

When you have two resistors in parallel, the total resistance is more complicated. For example consider Figure (2)

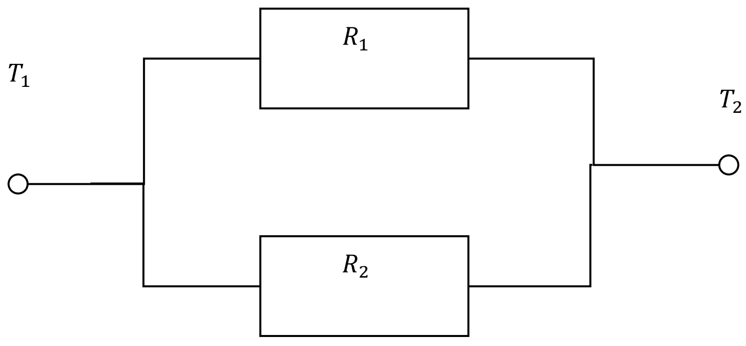

Figure 2: Two resistors in parallel

The Temperature difference across the two resistors must be the same in this case and the total heat flow is the sum of the heat flow in all branches;

$$T_2 - T_1 = Q_1R_1 = Q_2R_2 = QR$$

And

$$Q = Q_1 + Q_2$$

So

$$Q = \frac{\Delta T}{R} = \frac{\Delta T}{R_1} + \frac{\Delta T}{R_2} \rightarrow \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

So the inverse of the total resistance is the sum of the inverses of the resistances.

(3) Combinations of resistances with a single path.

Take the following example;

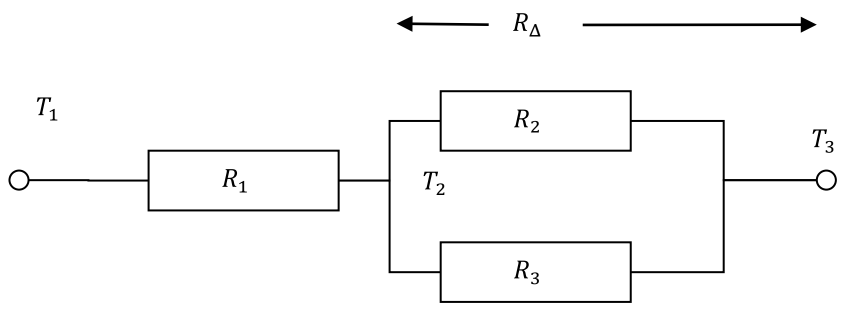

Figure 3: combined resistances in series and parallel

If we say that the two resistances in parallel have a combined resistanceR_Δ, then the total resistance is given by;

$$R = R_1 + R_\Delta$$

Where

$$R_\Delta = \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}}$$

Many people try to do everything in one line and most marks are lost in exams when they do it wrong. It is much better to break problems down into smaller chunks and to solve them slowly and accurately rather than rush it and make mistakes.

(4) General resistance networks through nodal analysis

In all the examples above there is a flow of heat from one side to another. While this is all you've experienced so far, there are other cases that are more general. For example the heat in a room flows out through the walls, ceiling and floor and there is usually more than one source of heat in a room. This means that many problems do not fall into the neat categories that you see for (1)-(3) above. Take for example the case where there is one source and two sinks of energy as in figure (4).

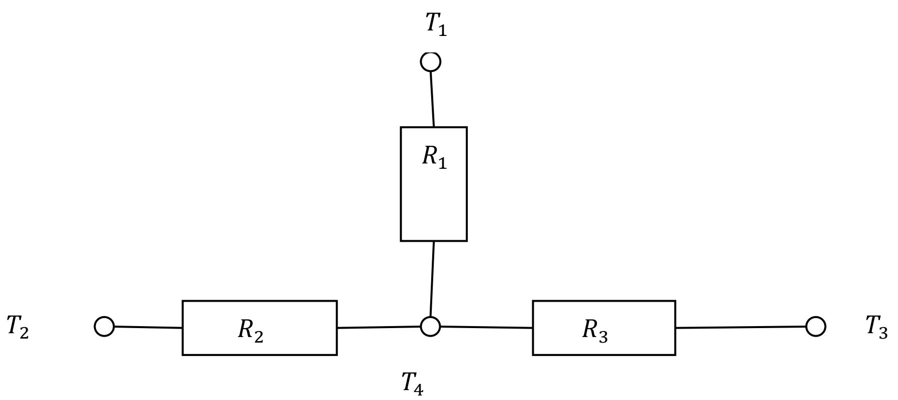

Figure 4: A general resistance network

This cannot be solved by the simple techniques outlined above. You need to use nodal analysis to solve these. Nodal analysis states that the sum of the flow of heat away from (or into) any particular node is always zero unless heat is being created (or destroyed) at the node.

This means that $Q_1 + Q_2 + Q_3 = 0$.

In terms of temperatures and resistances this can be expressed as:

$$\frac{T_2 - T_4}{R_2} + \frac{T_1 - T_4}{R_1} + \frac{T_3 - T_4}{R_3} = 0$$

This can be used to solve ANY resistance network, by solving the above equation for each node.